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SIMULATION OF LINEARIZED DYNAMICS OF GAS-TURBINE

ENGINES

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SUMMARY

Through the use of an electronic analog computer in the simulation of controlled aircraft-engine performance, one method of engine simulation has proved to be preferable from many considerations.

The equations used in developing this method of simulating the dynamics of gas-turbine engines are derived in general form from engine functional relations. This general simulation method can be utilized in the consideration of any first-order linear system and is designed for use in conjunction with control components for small perturbation or stability studies of controlled system operation.

A simulation of the response of a turbojet engine to a step change in an independent variable is made, and comparison of the experimental and simulated results indicates the validity of the simulation method presented.

Limitations on the use of altitude and flight-speed generalization factors in determining the coefficients necessary for the simulation of engine dynamics are discussed.

INTRODUCTION

A considerable amount of present-day control-system design and analysis is accomplished through the use of simulation techniques. With such techniques, part or all of a physical system is replaced by its mathematical representation, usually in the form of an analog computer facility. The mathematical representation then is used in subsequent examinations of controlled system behavior (reference 1).

The first step in the process of simulation is the determination of equations descriptive of the behavior of the system under all conditions of operation. The second step requires that these equations be put into a form applicable to the computer or simulation facilities

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available. Although the details of setting up a problem will vary considerably because of the wide variety of such machines, the same general equations are used in all cases.

An electronic analog computer has been used for some time at the NACA Lewis laboratory for simulation of aircraft-engine performance and studies of controlled engine operation. During the course of these activities, one method of simulation has proved to be the most advantageous in economy of computer facilities, utilization of experimental data, and accuracy of results. The object of this paper is to develop and present this method and to show its applicability to several types of gas-turbine engine.

BASIC DYNAMICS OF GAS-TURBINE ENGINES

For some given equilibrium condition, there will be established definite values of engine rotational speed, temperatures, pressures, and torques. At an equilibrium condition, the torque produced by the turbine will be totally absorbed by the compressor. If the equilibrium is disturbed by the injection of additional fuel, the instantaneous torque produced by the turbine will be greater than that absorbed by the compressor, and the engine will accelerate to a new equilibrium condition. For most practical cases, the rate of speed increase is proportional to the difference in turbine and compressor torques. This condition is typical of first-order systems. Consequently, exponential equations may be employed to describe the dynamic operation.

For description of the dynamic operation, a term is required which will determine the time history of the engine responses. In view of the exponential nature of the engine behavior, these responses can be described by specifying a time constant. This time constant is a ratio between the energy storage and energy dissipative elements and is the time required for 63 percent of a change to occur. It is a transient term and requires transient data for its evaluation.

The extent to which a variation in fuel flow will change an output variable such as engine rotational speed is the gain, or sensitivity of the engine rotational speed to changes in fuel flow. At any operating point, this gain is evaluated as the slope of the engine speed - fuel-flow curve and is the partial derivative $\frac{\partial N_e}{\partial W}$ evaluated with all independent variables other than fuel flow held constant. In a particular engine there are gain terms relating changes of rotational speed to changes in fuel flow, exhaust-nozzle area, and propeller blade angle; there are also gains which relate changes in

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pressures, temperatures, and torques to these same variations in fuel flow, exhaust-nozzle area, and blade angle. Gains related to changes in inlet geometry, guide-vane angle, and other independent variables also may be considered. The number of these terms which must be considered will depend upon the type of engine and the particular analysis or study undertaken. The gain terms will determine the quantitative change in a dependent or output variable produced by the variation of an independent or input variable. It will give no idea of the rate at which this change occurs.

To obtain the time relation of the various parameters, certain partial derivatives, evaluated with all independent variables held constant, must be obtained. These terms generally require the use of transient data for their determination, and they are specifically referred to in later sections of this report. All the gain terms and the time constant can be treated with standard altitude and flight-speed generalization factors.

Linear analysis was used in the development of this method of simulation. Although some broad simplifying assumptions must be made to allow use of linear analysis, it is possible to obtain reasonably accurate results with a minimum of mathematical complications. It has been demonstrated by comparison of analytic results with experimental performance that linear analysis is adequate to permit prediction of stability of engine and controls systems and to predict the response of these systems to small perturbations (reference 2). An example of particular importance in the use of linear analysis is the study of afterburner effects on the primary controlled engine.

DESCRIPTION OF ANALOG COMPUTER

The method of simulation of gas-turbine engines that is discussed in this report was developed for use on the electronic analog computer at the NACA Lewis laboratory. This computer is composed of operational amplifiers as indicated in figure 1. With a transfer function defined as the ratio of the Laplace transform of the output variable to the Laplace transform of the input variable, the approximate transfer functions of the operational amplifier is (reference 3)

$$\frac{y(s)}{x(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

(Symbols are defined in appendix A.) By selection of suitable pure resistive impedances, amplifiers may be constructed which will change algebraic sign, multiply by constant-valued coefficients, add, and subtract. If a capacitor C_2 is utilized for impedance Z_2 and a

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resistive element R_1 for impedance Z_1 , the amplifier unit yields as an output a time integral of the input variable. The resulting unit will have a time constant equal to R_1C_2 . A number of computational elements have been assembled to form the complete computer shown in figure 2.

Of special interest in connection with this report is the so-called matrix unit, composed of sign-changing, coefficient, and summation units, which yields the following relation between input and output variables:

$$y_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4$$
 $y_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4$
 $y_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4$
 $y_4 = a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4$

A diagrammatic representation of this unit is shown in figure 3.

Time behavior of variables is displayed on oscilloscopes and can be recorded by photographic techniques.

DEVELOPMENT OF SIMULATION METHOD

Turbine-propeller-engine simulation. - Development of the equations describing dynamic response of gas-turbine engines is based upon the assumption that thermodynamic and flow processes are quasi-static; that is, these processes continuously progress from one state of equilibrium to another on an equilibrium curve (reference 4). This assumption permits the writing of functional relations between the various input and output variables.

In analysis of turbine-propeller engines, the most useful relations are those between engine rotational speed, turbine-outlet temperature, fuel flow, and propeller blade angle. For analytic purposes, the accelerating torque is considered also. This torque is the difference between the torque produced by the engine and that absorbed by the propeller. The engine torque is a function of fuel flow and engine rotational speed:

$$Q_e = f_1(W,N_e) \tag{1}$$

The propeller torque is a function of propeller blade angle and propeller rotational speed:

$$Q_{p} = f_{2}(\beta, N_{p})$$
 (2)

The unbalanced torque referred to the engine shaft is the difference between engine and propeller torques, $(Q_e - \frac{Q_p}{r})$, where r is the gear ratio.

The turbine-outlet temperature is taken to be a function of fuel flow and engine rotational speed:

$$T = f_3(W,N_e)$$
 (3)

In a similar manner, any other engine-output variable may be expressed as a function of fuel flow and engine rotational speed.

Engine rotational speed is related to the unbalanced torque by the expression

$$\Delta N_{e} = \frac{1}{\hat{L}_{t}} \int \Delta Q \, dt \tag{4}$$

where I_{t} is the sum of the polar moment of inertia of the engine and the polar moment of inertia of the propeller referred to the engine.

Functions (1) and (2) may be expanded and linearized to give

$$\Delta Q_{e} = \frac{\partial Q_{e}}{\partial W} \left|_{N_{e}} \Delta W + \frac{\partial Q_{e}}{\partial N_{e}} \right|_{W} \Delta N_{e}$$
 (5)

$$\Delta Q_{\mathbf{p}} = \frac{\partial Q_{\mathbf{p}}}{\partial Q_{\mathbf{p}}} \bigg|_{\mathbf{N}_{\mathbf{p}}} \Delta \mathbf{p} + \frac{\partial \mathbf{N}_{\mathbf{p}}}{\partial Q_{\mathbf{p}}} \bigg|_{\mathbf{p}} \Delta \mathbf{N}_{\mathbf{p}}$$
 (6)

where $\frac{\partial Q_e}{\partial W}\Big|_{N_e}$ is the partial derivative of engine torque with respect to fuel flow, with engine speed held constant. Other terms are to be interpreted in similar manner.

The accelerating torque is then

Equation (3) is expanded and linearized to yield

$$\nabla \mathbf{I} = \frac{\partial \mathbf{M}}{\partial \mathbf{I}} \Big|_{\mathbf{M}^{\mathbf{G}}} + \frac{\partial \mathbf{M}^{\mathbf{G}}}{\partial \mathbf{I}} \Big|_{\mathbf{M}^{\mathbf{G}}}$$
(8)

The values of the terms forming the coefficients may be found from the slopes of steady-state operating curves, evaluated at particular values of fuel flow, blade angle, engine rotational speed, altitude, and flight speed. These equations together with equation (4) are introduced into the computer using the matrix unit described in a previous section. The interconnection diagram is shown in figure 4(a). This simulation will give an adequate description of dynamic operation of turbine-propeller engines over considerable ranges of operation (reference 5).

Turbojet-engine simulation. - Development of equations describing dynamic operation of turbojet engines requires the basic assumption of quasi-static operation, just as required in the case of turbine-propeller engines.

The input variables of turbojet engines are fuel flow and exhaustnozzle area. The output variables generally considered in controls
analysis are engine rotational speed and turbine-outlet temperature.
Another variable is necessary to account for engine operation at nonequilibrium conditions. The variable considered is accelerating
torque, which is the difference between the torque produced by the
turbine and that absorbed by the compressor.

The relations assumed between the engine variables are

$$Q = f_3(W,A,N_e)$$
 (9)

and

$$T = f_4(W,A,Q) \tag{10}$$

Under steady-state conditions, unbalanced torque is zero and engine rotational speed is an implicit function of fuel flow and exhaust-nozzle area. These functions may be expanded and linearized about some equilibrium point to yield

$$\Delta Q = \frac{\partial Q}{\partial W} \Big|_{A, N_e} \Delta W + \frac{\partial Q}{\partial A} \Big|_{W, N_e} \Delta A + \frac{\partial Q}{\partial N_e} \Big|_{W, A} \Delta N_e$$
 (11)

$$\Delta T = \frac{\partial T}{\partial A} |_{A,Q} + \frac{\partial T}{\partial A} |_{A,Q} + \frac{\partial T}{\partial A} |_{A,Q} + \frac{\partial T}{\partial A} |_{A,Q}$$
(12)

To evaluate these coefficients in this form would require plots of accelerating torque as a function of fuel flow, engine rotational speed and temperature as functions of accelerating torque, and temperature as a function of fuel flow. There is, however, no direct way of measuring this torque in a turbojet engine; consequently, to obtain the required plot would necessitate either a considerable amount of data processing or impractical test procedures. It is possible to circumvent these difficulties by manipulating the torque equation (11) in the following manner:

$$\frac{\partial Q}{\partial N} = \frac{\partial W}{\partial N} |_{W,A} + \frac{\partial W}{\partial N} |_{W,N_e} \Delta M + \Delta N_e$$

But

$$\frac{\frac{\partial Q}{\partial W}|_{A, N_e}}{\frac{\partial Q}{\partial N_e|_{W, A}}} = -\frac{\partial N_e}{\partial W}|_{A}$$

and

$$\frac{\frac{\partial Q}{\partial A}|_{W,N_e}}{\frac{\partial Q}{\partial N_e}|_{W,A}} = -\frac{\partial N_e}{\partial A}|_{W}$$

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Therefore,

$$-\frac{\partial Q}{\partial Q}\Big|_{W,A} = \frac{\partial W}{\partial W}\Big|_{A} + \frac{\partial W}{\partial A}\Big|_{W} + \frac{\partial W}{\partial A}\Big|_{W} - \Delta W_{e}$$
(13)

This manipulation has yielded terms whose coefficients may be evaluated from the slopes of the steady-state plots of speed against fuel flow and area. The slopes are evaluated at the appropriate operating points.

Equation (13) is multiplied by $\frac{\partial Q}{\partial N_e}|_{W,A}$ and substituted in equa-

tion (12) in this manner:

$$\nabla \mathbf{L} = \frac{9\mathbf{M}}{9\mathbf{L}} \begin{vmatrix} \mathbf{V} & \mathbf{V} & \mathbf{V} & \mathbf{V} & \mathbf{V} \\ \nabla \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{K}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} & \mathbf{V} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M} & \mathbf{V} \\ \mathbf{M} & \mathbf{V} \end{vmatrix} = \frac{9\mathbf{M}}{\mathbf{M}} \begin{vmatrix} \mathbf{M$$

But

$$\left. \frac{\partial Q}{\partial T} \right|^{M, V} \left(-\left. \frac{\partial N^{e}}{\partial S} \right|^{M, V} \right) = -\left. \frac{\partial L}{\partial N^{e}} \right|^{M, V}$$

Furthermore, in steady state, Q = 0, hence

$$\frac{\partial \mathbf{T}}{\partial \mathbf{W}} \Big|_{\mathbf{A}, \mathcal{Q}} = \frac{\partial \mathbf{T}}{\partial \mathbf{W}} \Big|_{\mathbf{A}}$$

and

$$\frac{\Delta T}{\Delta A} \bigg|_{W_2Q} = \frac{\Delta T}{\Delta A} \bigg|_{W}$$

Both of these terms are obtainable from steady-state data.

Thus, the equation of the response of turbine-outlet temperature is

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$$\Delta T = \frac{\partial T}{\partial W} \Big|_{A}^{\Delta W} + \frac{\partial T}{\partial A} \Big|_{W}^{\Delta A} - \frac{\partial T}{\partial N_{e}} \Big|_{W,A} \left(-\frac{\Delta Q}{\frac{\partial Q}{\partial N_{e}}} \Big|_{W,A} \right)$$
(14)

The torque-speed relations of a turbojet engine also may be expressed by equation (4). This equation is multiplied and divided by $\frac{\partial Q}{\partial N_e}$ in the following manner:

$$\Delta N_{e} = \left(-\frac{\frac{\partial Q}{\partial N_{e}}|_{W,A}}{I} \right) \frac{1}{s} \left(-\frac{\frac{\partial Q}{\partial N_{e}}|_{W,A}}{\frac{\partial Q}{\partial N_{e}}|_{W,A}} \right)$$

The engine time constant is defined $\tau_e \equiv -\frac{T}{\frac{\partial Q}{\partial N_e}|_{W,A}}$ and substituted

in the preceding equation to yield the expression

$$\Delta N_{e} = \frac{1}{\tau_{e}s} \left(-\frac{\Delta Q}{\frac{\partial Q}{\partial N_{e}}|_{W,A}} \right)$$
 (15)

The engine time constant, as well as $\frac{\partial T}{\partial N_e}\Big|_{W,A}$, is a typically

transient term and requires transient tests for its evaluation.

Equations (13) to (15) will determine speed and temperature response to changes in fuel flow. These are the equations used in simulation of turbojet engines. An interconnection diagram showing a method of using computer components for solution of these equations is shown in figure 4(b). (Terms in any column operate on the respective input to that column, and outputs are summed across the rows.)

Simulation of additional variables. - If additional input variables are to be considered, equations (9) and (10) must be extended in this manner:

$$Q = f_5(W,A,N_{\Theta}, ... X)$$

$$T = f_6(W,A,Q,...X)$$

where X denotes any additional input variable, such as variable inlet diffuser geometry, for example.

Functional relations of additional output variables can be formed by the same basic reasoning as that used for the temperature relations. If any additional output variable (compressor-discharge pressure, for example) is denoted as G, then

$$G = f_2(W,A,Q, \dots X)$$

which, upon suitable expansion, becomes similar in form to equation (14).

The manner in which the basic method may be extended to include simulation of additional engine variables in orderly fashion is illustrated in figure 5 (reference 6).

The manner in which the equations are evolved results in an array of equations wherein a maximum number of terms can be evaluated from steady-state data. All coefficients are slopes of steady-state curves, except the engine time constant $\tau_{\rm e}$ and the terms in the right column (fig. 5). Methods of determining the required values of terms with the minimum amount of testing are discussed in reference 7.

SAMPLE SIMULATION

The method shown in figure 4(b) was used to simulate the response of turbine-outlet temperature and engine rotational speed of a turbojet engine to an approximate step change in fuel flow. The simulated responses are shown in figure 6, and the comparable experimental responses are shown in figure 7. In this engine-dynamics simulation it was necessary to include dynamics of fuel system and test facilities. The fuel system was simulated by two first-order time lags and an interposed clipping network whose response duplicated the time behavior of fuel flow; the speed and temperature sensors were simulated by first-order time lags. The altitude test facilities were such that during transient operation engine-inlet pressure variations could not be eliminated, and thus a variation in nominal flight speed was produced. Compensation for this extraneous effect was introduced by evaluating gains and time constants directly from oscillograms that were to be simulated.

In figure 8 the simulated and experimental responses are compared. The degree of correlation indicates the validity of the description and simulation of the engine dynamic behavior.

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All experimental data obtained to date indicate that turbojet engines remain first order and linear for small perturbations over a wide range of flight conditions. Therefore, the developed method of simulation permits the exploration of a wide region of engine operation, provided only that the coefficients of the equations can be evaluated. If a complete set of equation coefficients can be obtained for any one flight condition, the coefficients may be adjusted for use at other flight conditions through the use of standard generalization factors. Complete generalization of engine dynamics should not be done indiscriminately, however. The fact that some knowledge must first be obtained of the altitude effects on the particular engine considered is discussed in appendix B.

CONCLUDING REMARKS

An electronic analog computer has been used for some time at the NACA Lewis laboratory in simulation of controlled aircraft-engine performance. One method of engine simulation has emerged as the most advantageous in economizing computer facilities, requiring the minimum amount of experimental data, and attaining the most accurate results.

The equations used in developing this method of simulating the dynamics of gas-turbine engines are derived in general form from engine functional relations. This general simulation method can be utilized in the consideration of any first-order system that can be linearized over a portion of its operating range. Any number of dependent and independent variables can be included in the analysis and simulation.

A simulation of the response of a turbojet engine to an approximate step change in fuel flow is made, and comparison of the simulated and experimental results indicates the validity of the simulation method. Since gas-turbine engines generally remain first order and linear for small perturbations over large operational regions, the simulation method can be utilized to explore engine characteristics over any range of flight conditions.

The use of altitude and flight-speed generalization factors in determining the equation coefficients necessary for the simulation of engine dynamics is discussed, and attention is drawn to the limitations placed on this use by the nature of component efficiency variation with flight condition.

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APPENDIX A

SYMBOLS

The following symbols are used in this report:

- A exhaust-nozzle area
- a constant (coefficient)
- C capacitance
- G additional output variable
- I polar moment of inertia
- K amplifier gain
- N rotational speed
- Q unbalanced (accelerating) torque
- R resistance
- r gear ratio, engine speed/propeller speed
- s complex Laplacian operator
- T absolute turbine-discharge temperature
- t time
- W fuel flow
- X additional input variable
- x arbitrary input signal
- y arbitrary output signal
- Z impedance
- β propeller blade angle
- Δ deviation from initial operating point
- η_{h} combustion efficiency
- τ time constant

Subscripts:

- e engine
- f final
- i initial
- p propeller
- t total

APPENDIX B

USE OF GENERALIZATION FACTORS IN EVALUATION

OF ENGINE DYNAMIC CHARACTERISTICS

The developed method of simulation permits the exploration of engine operation over a wide range of flight conditions, provided only that the coefficients of the equations can be evaluated.

Standard generalization factors have found widespread use in the handling of engine steady-state operational characteristics, since they allow prediction of engine performance in regions not covered by test. A similar use can be made of the generalization factors in the handling of dynamic characteristics. In the discussion of generalization Pactors it is noted that although many performance maps will generalize, those directly involving component efficiencies which vary greatly with flight condition will not do so. Fuel-flow relations are of the type having these large variations, and fuel is of primary interest with respect to engine controls. The generalization of engine dynamic terms, however, is not as limited as is the generalization of static characteristics. In some cases it has been illustrated that even though the absolute values of the fuel-flow relations would not generalize, the slopes of the relations (the dynamic characteristics) would do so. Such action indicates that even though the values or levels of efficiencies varied as flight conditions were changed, the form of the variation of efficiencies with rotational speed remained constant.

Some limitations must be placed on this use of generalization factors, however. For the engine with the dynamic characteristics shown in figure 7, it was found that the gain terms such as $\partial N/\partial W$ would not generalize. Figure 9 illustrates that the generalized gain terms varied as the altitude of operation changed. This variation indicates that not only efficiency levels but also the effect of rotational speed on component efficiencies changed with altitude.

The variation of combustion efficiency with flight condition, engine speed, and exhaust-nozzle area was determined; and compensation for this variation was added to the data previously presented in figure 9. Figure 10 shows that the gain terms of speed - fuel flow now will generalize. Other generalized gain terms $(\partial T/\partial W)$, for example) show similar improvement when compensation for combustion efficiency variations is introduced.

For the particular engine under consideration it appeared that efficiency changes in other components such as the compressor and turbine did not have a noticeable effect on the engine gain terms. The uncompensated gains could not be considered to generalize, but compensation for combustion efficiency variations alone was sufficient to obtain accurate results in determining the necessary gain terms of a turbojet engine at altitude.

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It therefore may be concluded that complete generalization of engine dynamics should not be done indiscriminately. Some knowledge must first be obtained of the altitude effects on the particular engine considered. If efficiencies vary consistently with altitude as did those considered in reference 4, complete generalization of dynamic terms may be used. For other engines sufficient altitude test data must first be obtained and the variations of efficiencies incorporated into the dynamic terms.

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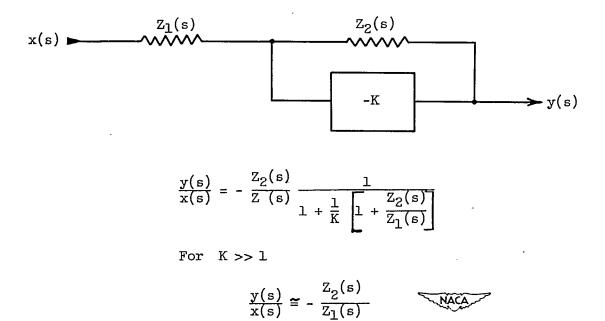


Figure 1. - Basic circuit used in electronic analog computer.

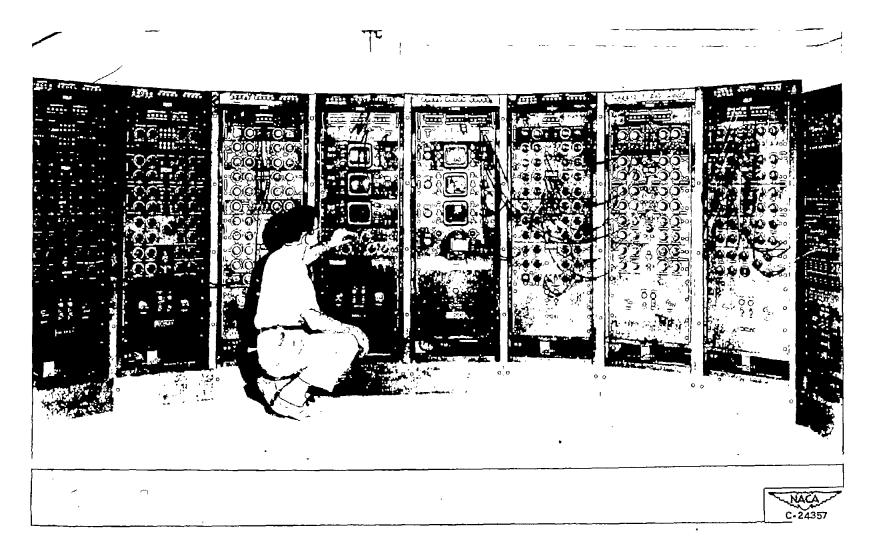


Figure 2. - Electronic analog computer used for simulation of dynamics of gas-turbine engines and investigation of controlled engine operation.

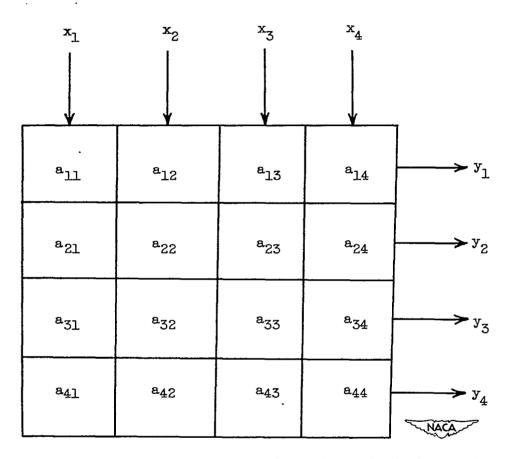
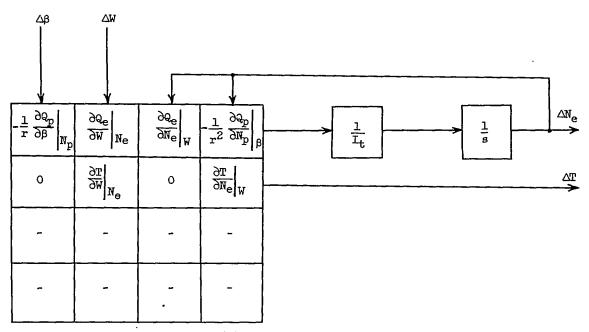
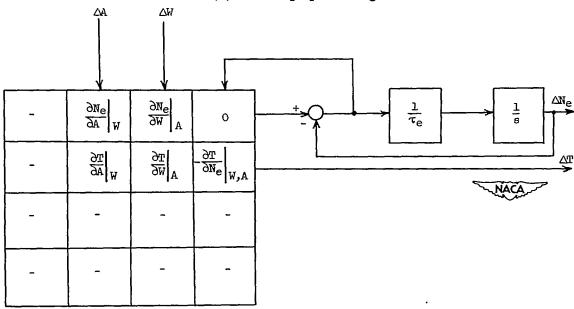


Figure 3. - Schematic diagram of matrix unit of electronic analog computer.



(a) Turbine-propeller engine.



(b) Turbojet engine.

Figure 4. - Interconnection diagram used in simulation of linearized dynamics of jet engines with electronic analog computer.

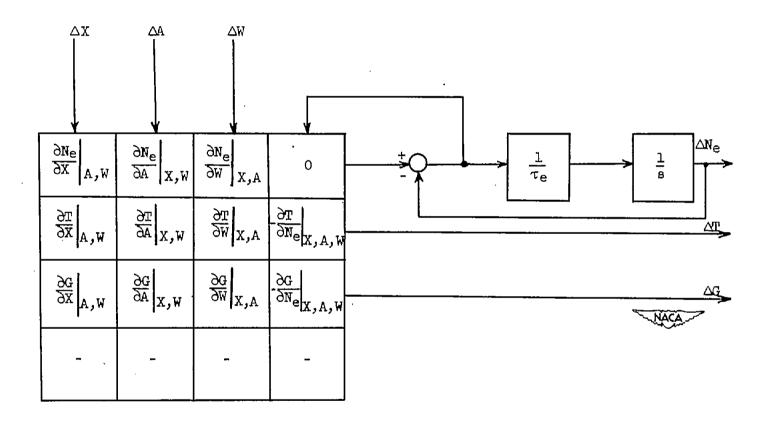


Figure 5. - Interconnection diagram used in simulation of linearized dynamics of turbojet engine showing manner in which additional variables can be simulated.

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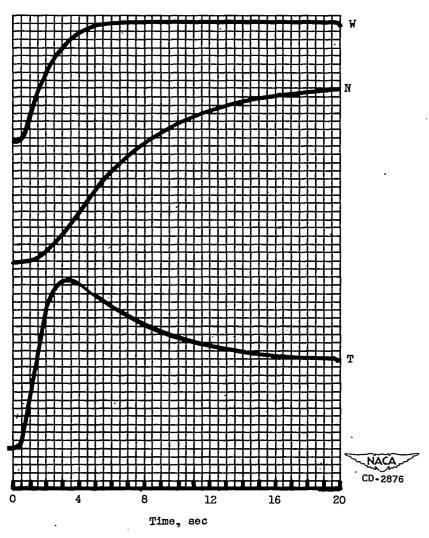


Figure 6. - Simulated speed and temperature responses of turbojet engine to change in fuel flow.

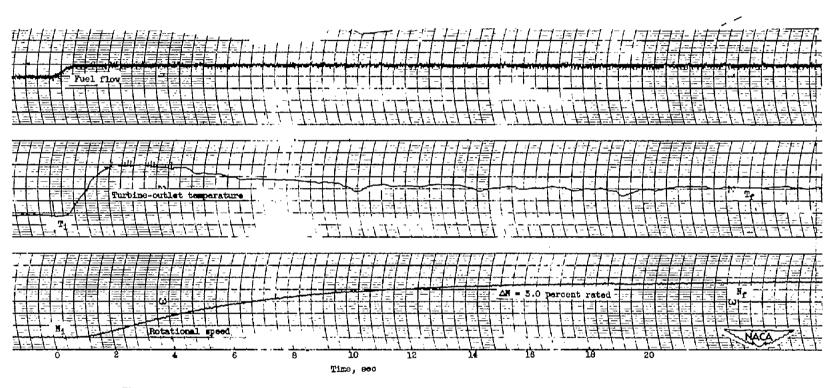


Figure 7. - Experimental response of turbojet engine to change in fuel flow at constant area and flight speed.

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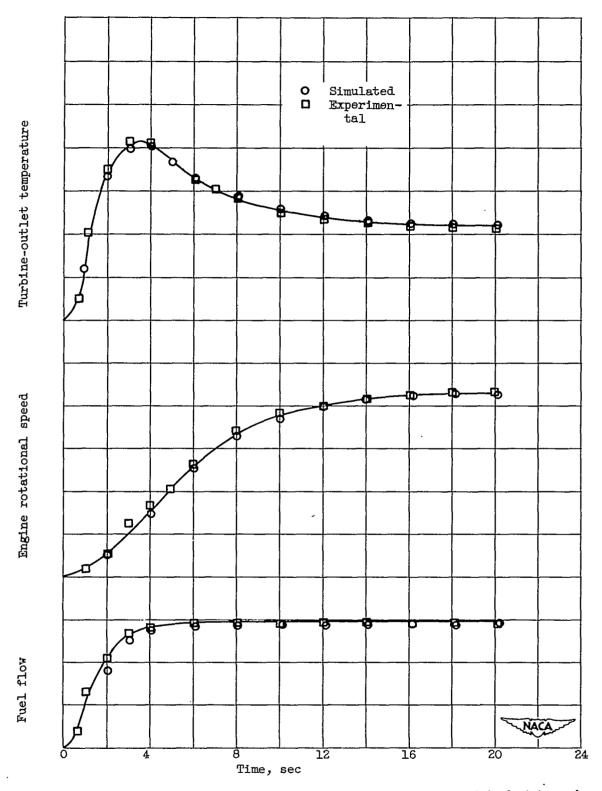


Figure 8. - Comparison of simulated and experimental responses of turbojet engine.

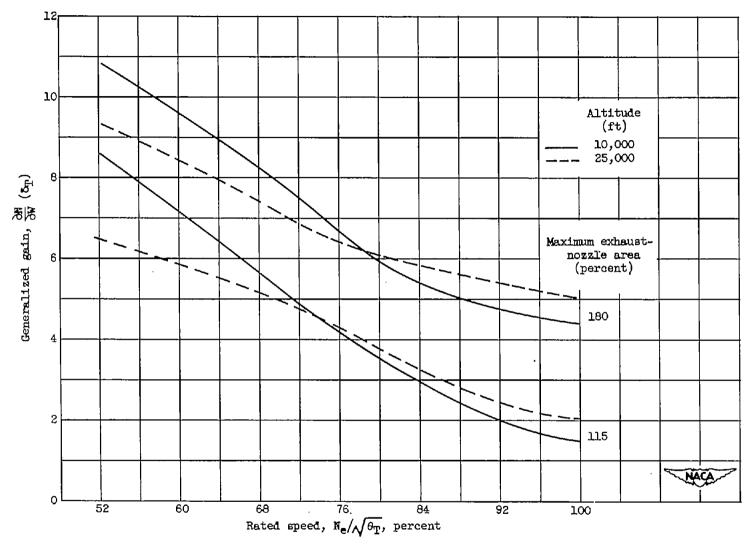


Figure 9. - Variation of generalized gain with engine rotational speed for turbojet engine at constant flight speed.

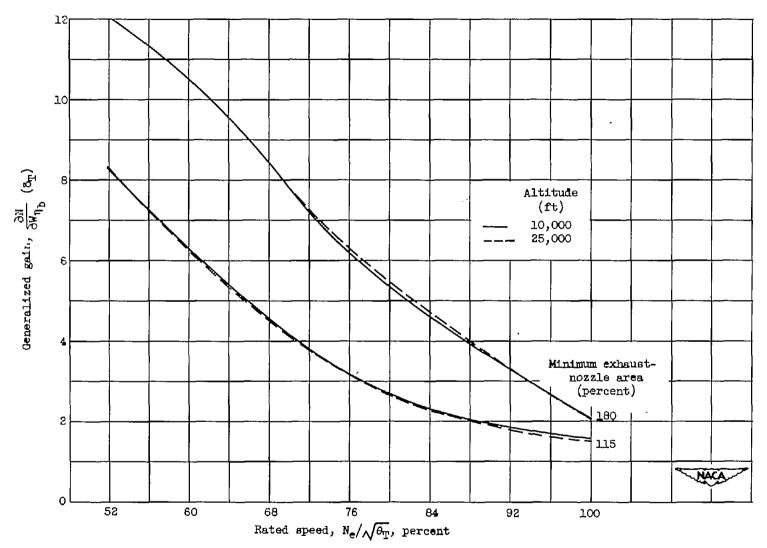


Figure 10. - Variation of generalized gain with engine rotational speed for turbojet engine. Gain compensated for combustion efficiency variations.